

## COMMON CORE STANDARDS WORKSHOP – MATH

### Kindergarten Counting & Cardinality: Math

- Already do
  1. Count to 100 by ones and by tens.
  2. Count forward beginning from a given number within the known sequence (instead of having to begin at 1).
  3. Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects). Understand the relationship between numbers and quantities; connect counting to cardinality.
  5. Count to answer “how many?” questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.
  6. Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.<sup>1</sup>
  7. Compare two numbers between 1 and 10 presented as written numerals.
- Differences
  - 1: we count by 2s and 5s
  - 3: we write to 100
  - The above skills are introduced but not required to master
- Aha!
  - Noticed we do more than standards require
- Question
  - What happens with students who do not have background with these standards now that we are eliminating pre-K?

### Kindergarten Operations/Algebraic

- Already do
  - Represent addition and subtraction with objects, fingers, mental images, drawings<sup>2</sup>, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.
  - Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.
  - For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.

-Fluently add and subtract within 5.

- Differences
  - 2: When certain teachers introduce ideas/standards according to different textbooks
  - 3, 4, 5: with supplemental material instruction: Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g.,  $5 = 2 + 3$  and  $5 = 4 + 1$ ).
- Aha!
  - No money or time in Kindergarten standards
  - Change report card with standards

#### Kindergarten Numbers & Operations Base Ten

- Already do
  - Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g.,  $18 = 10 + 8$ ); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.: through calendar & morning meeting
- Differences
  - No current assessment for the above.
- Aha!
  - Bring out old manipulatives for help with assessment

#### Kindergarten Measurement & Data

- Already do all of them!
- Aha!
  - Higher level kids are going to need more challenges according to these standards, or they will be bored!

#### Kindergarten Geometry

- Already do
  1. Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as *above*, *below*, *beside*, *in front of*, *behind*, and *next to*.

2. Correctly name shapes regardless of their orientations or overall size.
3. Identify shapes as two-dimensional (lying in a plane, “flat”) or three-dimensional (“solid”).
4. Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe the similarities, differences, parts (e.g., number of sides and vertices/“corners”) and other attributes (e.g., having sides of equal length).
5. Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes.
6. Compose simple shapes to form larger shapes. *For example, “Can you join these two triangles with full sides touching to make a rectangle?”*

- Differences
  - Vocabulary used with some standards
- Aha!
  - More materials for mastery
  - Not a standard for patterning

#### First Grade Operations/Algebraic

- Already do
  - Add/subtract with word problems
  - Apply properties of operations (not as much on associative)
  - Finding an unknown addend
  - Relating counting to addition and subtraction
  - Fact families (Saxon does not use many strategies)
- Differences
  - Adding three whole numbers in word problems
  - Decomposing a number
  - True and false
  - Determining unknown whole numbers other than sums of 10

#### First Grade Numbers & Operations Base Ten

- Already do
  - Count to 120
  - Understanding place value (Saxon needs to supplement with base tens)
  - Comparing two-digit numbers
  - Mentally adding/subtracting 10
  - Multiples of 10 computations
- Differences
  - Adding more two digit & one digit numbers
  - Reasoning & understanding are weak

## First Grade Measurement & Data

- Already do
  1. Order three objects by length; compare the lengths of two objects indirectly by using a third object.
  2. Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. *Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.*
  3. Tell and write time in hours and half-hours using analog and digital clocks.
  4. Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.

## First Grade Geometry

- Already do
  - 1. Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size) ; build and draw shapes to possess defining attributes.
  2. Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape.<sup>4</sup>
  3. Partition circles and rectangles into two and four equal shares, describe the shares using the words *halves*, *fourths*, and *quarters*, and use the phrases *half of*, *fourth of*, and *quarter of*. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.
- Questions/Aha!
  - Sometimes it is necessary to compose a ten – Does this mean regrouping?
  - It's impossible to cut money out of Saxon
  - Need more HOT with discussion
  - Needs to be beyond skill and drill
  - Odd & even numbers

## Second Grade Operations/Algebraic

- Already do
  1. Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting

together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

3. Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.

4. Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

- Differences
  - Mental strategies
  - Reinforcement in arrays
- Aha!
  - Wide variety of Saxon versions

## Second Grade Numbers & Operations Base Ten

- Already do
  1. Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:
    - a. 100 can be thought of as a bundle of ten tens — called a “hundred.”
    - b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).
  2. Count within 1000; skip-count by 5s, 10s, and 100s.
  4. Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using  $>$ ,  $=$ , and  $<$  symbols to record the results of comparisons.
  5. Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.
  6. Add up to four two-digit numbers using strategies based on place value and properties of operations.
  7. Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.
  8. Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900.

9. Explain why addition and subtraction strategies work, using place value and the properties of operations.<sup>3</sup>

- Differences
  - Adding four “2-digit” numbers
  - Extended response – different versions/Saxon

#### Second Grade Measurement & Data

- Already do all of them!
- No differences!

#### Second Grade Geometry

- Already do all of them!
- No differences!

#### Third Grade Operations/Algebraic

- Already do all of them!
- Differences
  - Need to go more in-depth on what we do
- Aha!
  - Multiplication sooner and more division
- Questions
  - How do we do this with those who have not mastered addition & subtraction?

#### Third Grade: Numbers & Operations Base Ten

- Already do all of them!
- Aha!
  - Multiplication sooner

#### Third Grade: Numbers & Operations Fractions

- Already cover it: compare, add, subtract with common denominators
- This is very in-depth & we don't come close to covering it enough for these.
- Aha!
  - We introduce these but don't master them

## Third Grade: Measurement & Data

- Already do
  1. Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.
  3. Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step “how many more” and “how many less” problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets.*
  4. Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters.
  5. Recognize area as an attribute of plane figures and understand concepts of area measurement.
    - a. A square with side length 1 unit, called “a unit square,” is said to have “one square unit” of area, and can be used to measure area.
    - b. A plane figure which can be covered without gaps or overlaps by  $n$  unit squares is said to have an area of  $n$  square units.
  6. Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).
  7. Relate area to the operations of multiplication and addition.
    - a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.
    - b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.
    - c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths  $a$  and  $b + c$  is the sum of  $a \times b$  and  $a \times c$ . Use area models to represent the distributive property in mathematical reasoning.
    - d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.
- Differences
  - Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l).6 Add,

subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem

- Aha!  
-Study island helps supplement these standards

#### Third Grade: Geometry

- Already do all of them!

#### Fourth Grade: Operations/Algebraic

- Already do
  1. Interpret a multiplication equation as a comparison, e.g., interpret  $35 = 5 \times 7$  as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.
  2. Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.<sup>1</sup>
  3. Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.
- Differences
  - Prime & composite
  - Terminology
  - Getting to all of the material

#### Fourth Grade: Numbers & Operations Base Ten

- Already do
  1. Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that  $700 \div 70 = 10$  by applying concepts of place value and division.*
  2. Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using  $>$ ,  $=$ , and  $<$

symbols to record the results of comparisons.

3. Use place value understanding to round multi-digit whole numbers to any place.

4. Fluently add and subtract multi-digit whole numbers using the standard algorithm.

5. Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

6. Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

- Difference
  - More written explanation
- Question
  - How in-depth rectangular array?

#### Fourth Grade: Numbers & Operations Fractions

- Already do
  - Explain why a fraction  $a/b$  is equivalent to a fraction  $(n \times a)/(n \times b)$  by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.
- Difference
  - Do earlier; more intending to teach; common denominators, more with models: Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as  $1/2$ . Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols  $>$ ,  $=$ , or  $<$ , and justify the conclusions, e.g., by using a visual fraction model.
  - Understand a multiple of  $a/b$  as a multiple of  $1/b$ , and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express  $3 \times (2/5)$  as  $6 \times (1/5)$ , recognizing this product as  $6/5$ . (In general,  $n \times (a/b) = (n \times a)/b$ .)*
  - Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. *For example, express  $3/10$  as  $30/100$ , and add  $3/10 + 4/100 = 34/100$ .*

- Aha!  
-Teach more fractions!
- Question  
-What do we need to know for each benchmark?

#### Fourth Grade: Measurement & Data

- Already do
  1. Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a twocolumn table. *For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...*
  2. Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.
  3. Apply the area and perimeter formulas for rectangles in real world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.*
- Differences
  - Line plots
  5. Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement
  6. Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.
  7. Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.

#### Fourth Grade: Geometry

- Already do it all!

#### Fifth Grade: Operations & Algebraic Thinking

- Numeral Expression
  - Saxon: Yes
  - Houghton/Mifflin: Yes
  - McMillan/McGraw: Yes
  - Scott Foresman: Yes
- Patterns & relationships
  - Saxon: Yes
  - Houghton/Mifflin: Yes
  - McMillan/McGraw: Yes
  - Scott Foresman: Yes

#### Fifth Grade: Numbers & Operations Base Ten

- Understanding Place Value
  - Saxon: Yes
  - Houghton/Mifflin: Yes
  - McMillan/McGraw: Yes
  - Scott Foresman: Yes
- Performing Operations
  - Saxon: No
  - Houghton/Mifflin: Yes
  - McMillan/McGraw: Yes
  - Scott Foresman: Yes

#### Fifth Grade: Numbers & Operations Fractions

- Addition & Subtraction
  - Saxon: 1. Yes; 2. No
  - Houghton/Mifflin: 1. Yes; 2. No
  - McMillan/McGraw: 1. Yes; 2. No
  - Scott Foresman: 1. Yes; 2. Yes
- Multiply & Divide
  - Saxon: 3. Yes; 4. Yes; 5. No; 6. No; 7. No
  - Houghton/Mifflin: 3. Yes; 4. Yes; 5. No; 6. Yes; 7. Yes
  - McMillan/McGraw: 3. Yes; 4. Yes; 5. Yes; 6. Yes; 7. Yes
  - Scott Foresman: 3. Yes; 4. Yes; 5. Yes; 6. Yes; 7. Yes

## Fifth Grade: Measurement & Data

- Convert
  - Saxon: No
  - Houghton/Mifflin: No
  - McMillan/McGraw: Yes
  - Scott Foresman: Yes
- Represent
  - Saxon: No
  - Houghton/Mifflin: No
  - McMillan/McGraw: Yes
  - Scott Foresman: Yes
- Understand geometry
  - Saxon: 3. No; 4. No; 5. No
  - Houghton/Mifflin: 3. No; 4. No; 5. No
  - McMillan/McGraw: 3. Yes; 4. Yes; 5. Yes
  - Scott Foresman: 3. Yes; 4. Yes; 5. Yes

## Fifth Grade: Geometry

- Graph
  - Saxon: 1. Yes; 2. Yes
  - Houghton/Mifflin: 1. Yes; 2. Yes
  - McMillan/McGraw: 1. Yes; 2. Yes
  - Scott Foresman: 1. Yes; 2. Yes
- Classify
  - Saxon: 1. Yes; 2. No
  - Houghton/Mifflin: 1. Yes; 2. Yes
  - McMillan/McGraw: 1. Yes; 2. Yes
  - Scott Foresman: 1. Yes; 2. Yes

## Sixth Grade: Math, Ratios and Proportional Relationships

- Already do
  1. Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, “The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak.” “For every vote candidate A received, candidate C received nearly three votes.”*
  2. Understand the concept of a unit rate  $a/b$  associated with a ratio  $a:b$  with  $b \neq 0$ , and use rate language in the context of a ratio relationship.

*For example, “This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is  $\frac{3}{4}$  cup of flour for each cup of sugar.” “We paid \$75 for 15 hamburgers, which is a rate of \$5 per hamburger.”<sup>1</sup>*

3. Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*

c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means  $\frac{30}{100}$  times the quantity); solve problems involving finding the whole, given a part and the percent.

- Differences

-3. Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

a. Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

- Aha!

-How do we teach depth with Saxon?

## Sixth Grade: Math, The Number System

- Already do

1. Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for  $(\frac{2}{3}) \div (\frac{3}{4})$  and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that  $(\frac{2}{3}) \div (\frac{3}{4}) = \frac{8}{9}$  because  $\frac{3}{4}$  of  $\frac{8}{9}$  is  $\frac{2}{3}$ . (In general,  $(\frac{a}{b}) \div (\frac{c}{d}) = \frac{ad}{bc}$ .) How much chocolate will each person get if 3 people share  $\frac{1}{2}$  lb of chocolate equally? How many  $\frac{3}{4}$ -cup servings are in  $\frac{2}{3}$  of a cup of yogurt? How wide is a rectangular strip of land with length  $\frac{3}{4}$  mi and area  $\frac{1}{2}$  square mi?*

2. Fluently divide multi-digit numbers using the standard algorithm.

3. Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

5. Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g.,

temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.

6. Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.

a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g.,  $-(-3) = 3$ , and that 0 is its own opposite.

c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.

7. Understand ordering and absolute value of rational numbers.

a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret  $-3 > -7$  as a statement that  $-3$  is located to the right of  $-7$  on a number line oriented from left to right.*

b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write  $-3\text{ }^{\circ}\text{C} > -7\text{ }^{\circ}\text{C}$  to express the fact that  $-3\text{ }^{\circ}\text{C}$  is warmer than  $-7\text{ }^{\circ}\text{C}$ .*

- Differences

4. Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express  $36 + 8$  as  $4(9 + 2)$ .*

6. Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.

b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.

7. Understand ordering and absolute value of rational numbers.

c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of  $-$*

30 dollars, write  $|-30| = 30$  to describe the size of the debt in dollars.

d. Distinguish comparisons of absolute value from statements about order. *For example, recognize that an account balance less than  $-30$  dollars represents a debt greater than 30 dollars.*

8. Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

- Aha!  
-Real Talk – How/why do we use it?

### Sixth Grade: Math, Expressions & Equations

- Already do
  1. Write and evaluate numerical expressions involving whole-number exponents.
  2. Write, read, and evaluate expressions in which letters stand for numbers.
    - a. Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation “Subtract  $y$  from 5” as  $5 - y$ .*
    - b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression  $2(8 + 7)$  as a product of two factors; view  $(8 + 7)$  as both a single entity and a sum of two terms.*
- Differences
  3. Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression  $3(2 + x)$  to produce the equivalent expression  $6 + 3x$ ; apply the distributive property to the expression  $24x + 18y$  to produce the equivalent expression  $6(4x + 3y)$ ; apply properties of operations to  $y + y + y$  to produce the equivalent expression  $3y$ .*
  4. Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions  $y + y + y$  and  $3y$  are equivalent because they name the same number regardless of which number  $y$  stands for.*
  5. Understand solving an equation or inequality as a process of

answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.

6. Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.

7. Solve real-world and mathematical problems by writing and solving equations of the form  $x + p = q$  and  $px = q$  for cases in which  $p$ ,  $q$  and  $x$  are all nonnegative rational numbers.

8. Write an inequality of the form  $x > c$  or  $x < c$  to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form  $x > c$  or  $x < c$  have infinitely many solutions; represent solutions of such inequalities on number line diagrams.

9. Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation  $d = 65t$  to represent the relationship between distance and time.*

## Sixth Grade: Geometry

- Already do
  1. Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.
  2. Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas  $V = lwh$  and  $V = bh$  to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.
  3. Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.
- Differences
  - Represent three-dimensional figures using nets made up of rectangles

and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

#### Sixth Grade: Statistics & Probability

- Already do
  - Mean, median, mode
- Differences
  1. Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, “How old am I?” is not a statistical question, but “How old are the students in my school?” is a statistical question because one anticipates variability in students’ ages.*
  2. Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.
  3. Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.
  4. Display numerical data in plots on a number line, including dot plots, histograms, and box plots.
  5. Summarize numerical data sets in relation to their context, such as by:
    - a. Reporting the number of observations.
    - b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.
    - c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.
    - d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.
- Aha!
  - Where is probability?

#### Seventh Grade: Ratios and Proportional Relationships

- Already do
  1. Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks  $\frac{1}{2}$  mile in each  $\frac{1}{4}$  hour, compute*

*the unit rate as the complex fraction  $1/2/1/4$  miles per hour, equivalently 2 miles per hour.*

3. Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*

- Differences
  2. Recognize and represent proportional relationships between quantities.
    - d. Explain what a point  $(x, y)$  on the graph of a proportional relationship means in terms of the situation, with special attention to the points  $(0, 0)$  and  $(1, r)$  where  $r$  is the unit rate.
- Aha!
  - All classes need to go deeper
  - Text issues

## Seventh Grade: Number System

- Already do
  1. Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.
    - a. Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.*
    - b. Understand  $p + q$  as the number located a distance  $|q|$  from  $p$ , in the positive or negative direction depending on whether  $q$  is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.
    - c. Understand subtraction of rational numbers as adding the additive inverse,  $p - q = p + (-q)$ . Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.
    - d. Apply properties of operations as strategies to add and subtract rational numbers.
  2. Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.
    - a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as  $(-1)(-1) = 1$  and the rules

for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.

b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If  $p$  and  $q$  are integers, then  $-(p/q) = (-p)/q = p/(-q)$ . Interpret quotients of rational numbers by describing real world contexts.

c. Apply properties of operations as strategies to multiply and divide rational numbers.

d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.

3. Solve real-world and mathematical problems involving the four operations with rational numbers.1

### Seventh Grade: Equation & Expressions

- Already do
  1. Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.
- Differences
  2. Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example,  $a + 0.05a = 1.05a$  means that “increase by 5%” is the same as “multiply by 1.05.”*
  3. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making \$25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or \$2.50, for a new salary of \$27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.*
  4. Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

### Seventh Grade: Geometry

- Already do
  1. Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing

and reproducing a scale drawing at a different scale.

4. Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

5. Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.

6. Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

- Difference
  - Rotation & reflection taught – not core curriculum

#### Seventh Grade: Statistics & Probability

- Already do
  1. Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.
  2. Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.*
  3. Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.*
  4. Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.*
  5. Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event

that is neither unlikely nor likely, and a probability near 1 indicates a likely event.

6. Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.*

- Difference

-Students don't do independently:

7. Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.

8. Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.

#### Eighth Grade: Number System

- Already do

1. Understand informally that every number has a decimal expansion; the rational numbers are those with decimal expansions that terminate in 0s or eventually repeat. Know that other numbers are called irrational.

2. Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g.,  $\pi^2$ ). *For example, by truncating the decimal expansion of  $\sqrt{2}$ , show that  $\sqrt{2}$  is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*

#### Eighth Grade: Equations & Expressions

- Already do

1. Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example,  $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$ .*

2. Use square root and cube root symbols to represent solutions to equations of the form  $x^2 = p$  and  $x^3 = p$ , where  $p$  is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that  $\sqrt{2}$  is irrational.

3. Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as  $3 \times 10^8$  and the population of the world as  $7 \times 10^9$ , and determine that the world population is more*

*than 20 times larger.*

4. Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

5. Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*

- Aha!  
-We are doing a lot of what the core wants, we just need to go deeper.

#### Eighth Grade: Geometry

- Already do
  1. Verify experimentally the properties of rotations, reflections, and translations:
    - a. Lines are taken to lines, and line segments to line segments of the same length.
    - b. Angles are taken to angles of the same measure.
    - c. Parallel lines are taken to parallel lines.
  2. Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.
  3. Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.
  4. Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar twodimensional figures, describe a sequence that exhibits the similarity between them.
  5. Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*
  7. Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.
  8. Apply the Pythagorean Theorem to find the distance between two

points in a coordinate system.

- Differences
  - Can't explain a proof
  - Have volume formulas memorized

Eighth Grade: Statistics & Probability

- Already do
  - We do all skills at the foundation level, but not at the depth the core curriculum expects.

High School Math: Geometry

- Already do
  - We cover all the topics and the basics
- Differences
  - Students are expected to bring prior knowledge
- Aha!
  - The common core curriculum is our second semester curriculum.

High School Math: Algebra 1

- Already do
  - Exponents
  - Linear equations
  - Quadratic equations
  - Systems of equations: linear to linear
- Difference
  - Rational exponents
  - Exponential equations
  - Systems of equations: linear to quadratic

High School Math

- Already do
  - We teach the standards but we could always go deeper.
  - Similar content
  - Emphasis on process, not only the solution
- Differences

- More modeling
- Separate standards by categories, not grade levels
- Saxon?
- Geometry needs to hit all topics
- Advanced students have extra standards
- More focus on functions
- Graphing on polar plane
- Prove circles similar

- Aha!
  - Students create real-life problems
  - Teaching Algebra  $\frac{1}{2}$  in high school
  - Algebra 2 before geometry?
  - Easier to follow appendix
  - Standards are not listed by courses
  - Formal geometric proofs and constructions